Estimating model parameters by chaos synchronization

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Using chaos synchronization and a proposed iterative method of parameter adaption, we precisely estimate the model parameters of chaotic systems and synchronize two chaotic systems with originally mismatching model parameters. This parameter adaption method can be applied to a spatiotemporal chaotic system with a one-way-coupled map lattice. As a biomedical application, this method is capable of estimating the asymmetric tension parameter of a vocal fold model.

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Chaos synchronization has recently become subject of intensive study [1–5] and it has been applied to speech communication [6,7]. Despite the sensitivity of chaotic systems to initial conditions, two chaotic systems, when appropriately coupled, can be synchronized, opening up a new area of research. In practical applications of chaos synchronization, parameter mismatches inevitably exist, and these mismatches have been found to play an important role in the degradation of synchronization [1,3,8]. Thus, synchronizing two chaotic systems with originally mismatching parameters becomes significant. On the other hand, global and local techniques [9,10] have been used in recent years to reconstruct model parameters of chaotic systems from a time series [11]. Inevitable prediction errors decrease the applicability of these approximation methods to high-dimensional systems, particularly spatiotemporal chaotic systems. Chaos synchronization has provided researchers with techniques to synchronize high-dimensional systems, and it could be a promising method for precisely estimating model parameters [12–16].

In this paper, based on chaos synchronization, we propose an iterative method of parameter adaption to estimate model parameters. Adaptive parameter control can be determined by using a parameter difference map. The stable zero solution of this map demonstrates that two chaotic systems with an original parameter mismatch are finally synchronized, and their model parameters converge to the same results. We also used this parameter adaption method to estimate the model parameters of a spatiotemporal chaotic system with a one-way-coupled open map lattice (OCOML) and to synchronize spatiotemporal chaotic systems. One application of this method to biomedical systems is to estimate asymmetric tension parameters in a biomechanical model of vocal folds.

To study chaos synchronization and adaptive parameter control, we consider unidirectionally coupled systems, letting the drive system be

$$\dot{\mathbf{x}}=\mathbf{F}(\mathbf{p},\mathbf{x},s),\quad \mathbf{x}\in\mathbb{R}^m,\quad \mathbf{p}=\{p_1,p_2,\ldots,p_M\}^T \qquad (1)$$

with the driving function being $s=h(\mathbf{x})$ [2]. The response system with the same model equation but different model parameters can be described as

$$\dot{\mathbf{y}}=\mathbf{F}(\tilde{\mathbf{p}},\mathbf{y},s),\quad \mathbf{y}\in\mathbb{R}^m,\quad \tilde{\mathbf{p}}=\{\tilde{p}_1,\tilde{p}_2,\ldots,\tilde{p}_M\}^T. \qquad (2)$$

For small $\mathbf{e}=\mathbf{y}-\mathbf{x}$, the difference dynamics has the Taylor expansion:

$$\dot{\mathbf{e}}=\mathbf{F}_{\mathbf{y}}(\tilde{\mathbf{p}},\mathbf{y},s)\mathbf{e}+\sum_{i=1}^{M}\mathbf{F}_{\tilde{p}_i}(\tilde{p}_i,\mathbf{y},s)\Delta p_i + \frac{1}{2}\left(\mathbf{e}\frac{\partial}{\partial\mathbf{y}}+\sum_{i=1}^{M}\frac{\partial}{\partial\tilde{p}_i}\Delta p_i\right)^2\mathbf{F}(\tilde{p}_i,\mathbf{y},s)+\cdots, \qquad (3)$$

where $\Delta p_i=\tilde{p}_i-p_i$. When $\mathbf{p}=\tilde{\mathbf{p}}$, we consider that, using the scalar observable $s(t)$ constructed by the decomposition introduced by Pecora and Carroll [1] or by active-passive decomposition [2], Eq. (3) has a stable zero solution $\mathbf{e}_s\to 0$ with $t\to\infty$. As a result, the two systems are synchronous $\mathbf{x}=\mathbf{y}$. However, when $\Delta p_i\neq 0$, the synchronization error does not converge to zero. For significantly small parameter mismatches $|\Delta p_i/p_i|\ll 1$, the linearized Eq. (3) is integrated as

$$\mathbf{e}(t)=\int_0^t \mathbf{e}_s(t-\tau)\sum_{i=1}^{M}\mathbf{F}_{\tilde{p}_i}(\tilde{p}_i,\mathbf{y},s)\Delta p_i d\tau. \qquad (4)$$

Then, we describe the relationship between $\mathbf{e}$ and $\Delta\mathbf{p}$ as

$$\mathbf{e}=\mathbf{B}\Delta\mathbf{p}, \qquad (5)$$

where $\mathbf{e}=\{e_1,e_2,\ldots,e_m\}^T$ and $\Delta\mathbf{p}=\{\Delta p_1,\Delta p_2,\ldots,\Delta p_M\}^T$. $\mathbf{B}=\{\mathbf{B}_1,\mathbf{B}_2,\ldots,\mathbf{B}_M\}$ is the function of $\tilde{p}_i$, y, s. To obtain $\mathbf{B}_i$, the M additional subsystems with parameters $\tilde{\mathbf{p}}^{(i)}=\{\tilde{p}_1,\ldots,\tilde{p}_i+\Delta,\ldots,\tilde{p}_M\}^T$ are constructed by slightly perturbing the model parameters of the responser [15]. The state difference between the ith subsystem and the response system is $\mathbf{e}^{(i)}$. Then $\mathbf{B}_i$ can be approximately estimated by $\tilde{\mathbf{B}}_i=\mathbf{e}^{(i)}/\Delta$. Thus, for a time-evolution vector $\mathbf{e}(j\tau)$ with the time interval τ and length N, $\Delta\mathbf{p}$ can be approximated by using the least-squares fit $\Sigma_{j=0}^{N}\|\mathbf{e}(j\tau)-\tilde{\mathbf{B}}\Delta\mathbf{p}\|^2=$minimum as

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$$\Delta\tilde{\mathbf{p}} = (\tilde{\mathbf{B}}^T\tilde{\mathbf{B}})^{-1}\tilde{\mathbf{B}}^T\mathbf{e}. \tag{6}$$

$\tilde{\mathbf{B}}$ and $\Delta\tilde{\mathbf{p}}$ are approximations of $\mathbf{B}$ and $\Delta\mathbf{p}$ under $\Delta \ll 1$ and $|\Delta p_i/p_i| \ll 1$. Furthermore, for larger parameter mismatches, the high-order perturbations in Eq. (3) would produce the difference $\|\Delta\mathbf{B}\| = \|\tilde{\mathbf{B}} - \mathbf{B}\| \sim \delta$ (when $|\Delta p_i/p_i| \ll 1$, $\delta \ll 1$ is satisfied). Therefore, to approach the model parameter $\mathbf{p}$ of the drive system within a wider range of parameter mismatches, based on Eqs. (5) and (6), we here propose an iterative scheme of parameter adaption: we estimate the parameter difference $\Delta\tilde{\mathbf{p}}(k)$ using Eq. (6), and then adjust the response parameters using $\mathbf{p}(k+1) = \mathbf{p}(k) - \Delta\tilde{\mathbf{p}}(k)$ until $\Delta\tilde{\mathbf{p}}(k)$ approaches zero when iterative number k is significantly large. This iteration procedure can be formulated, and its convergence can be determined by the following parameter difference map:

$$\Delta\mathbf{p}(k+1) = \mathbf{P}(k)\Delta\mathbf{p}(k), \quad k = 1,2,\ldots, \tag{7}$$

where $\Delta\mathbf{p}(k) = \mathbf{p}(k) - \mathbf{p}$, $\mathbf{p}(1) = \tilde{\mathbf{p}}$, $\mathbf{P}(k) = [\tilde{\mathbf{B}}^T(k)\tilde{\mathbf{B}}(k)]^{-1}\tilde{\mathbf{B}}^T(k)\Delta\mathbf{B}(k)$, and $\Delta\mathbf{B}(k) = \tilde{\mathbf{B}}(k) - \mathbf{B}(k)$. For the bounded $[\tilde{\mathbf{B}}^T(k)\tilde{\mathbf{B}}(k)]^{-1}\tilde{\mathbf{B}}^T(k)$ and small perturbation $\|\Delta\mathbf{B}\| \sim \delta \ll 1$, $\|\mathbf{P}\| \ll 1$ and $\|\mathbf{P}^T\| \ll 1$ are yielded, and thus Eq. (7) has a stable zero solution, that is, $\lim_{k\to\infty}\Delta p_i^2(k) = 0$ when k is significantly large. Therefore, this iterative scheme of parameter adaption is effective when the original values of the response parameters are within a certain range. Otherwise, significantly large parameter mismatches may lead to an unstable zero solution of Eq. (7), and thus the parameter adaption would not work. By using this parameter adaption method, the parameters $\mathbf{p}(k)$ of the response system asymptotically converge to $\mathbf{p}$ of the drive system, so that two chaotic systems with an original parameter mismatch can be finally synchronized. This algorithm is applicable to both continuous and discrete time systems. In particular, Eq. (7) also can be deduced from the feedback method of chaos synchronization [3], suggesting its general applicability to chaos synchronization.

To show the iterative process of parameter adaption, we consider the first example using the Lorenz system: $\dot{x} = p_1(y-x)$, $\dot{y} = p_2 s - y - sz$, $\dot{z} = sy - p_3 z$, and the driving signal $s(t) = x$. When $p_i = \tilde{p}_i$, synchronization occurs with x driving [1]. The original model parameters (p_1, p_2, p_3) of the drive and response systems are (10,40,8/3) and (16,45.92,4), respectively. In Fig. 1(a), when parameter adaption is imposed, the response parameters converge to 10, 40, and 8/3, indicating that this technique allows us to estimate exactly the model parameters of the drive system. The time-averaged synchronization error $E = (1/N\tau)\int_0^{N\tau}\|\mathbf{x} - \mathbf{y}\|^2 dt$ decreases to zero, and two chaotic systems with an original parameter mismatch are finally synchronous [see Fig. 1(b), where $\tau = 0.01$, $N = 10^4$, and $\Delta = 10^{-4}$]. The iterative method of parameter adaption works even for a large original parameter mismatch, demonstrating its robustness (see Fig. 2, where $p_2 = \tilde{p}_2 = 40$). Once the parameter estimation has been achieved, synchronization is robust to external perturbations [1,16]. When the original values of $\tilde{p}_1$ and $\tilde{p}_3$ are chosen in the black region I of Fig. 2, the influences of high-order perturbation terms in Eq. (3) are small, and thus, Eq. (7) has a stable zero solution. The final results of $\tilde{p}_1$ and $\tilde{p}_3$ converge to 10 and 8/3, respectively, and synchronization is achieved. However, when the original values of $\tilde{p}_1$ and $\tilde{p}_3$ are out of this region (see the white region II), the large parameter and state differences lead to an unstable zero solution of Eq. (7). Thus, in region II, the parameters of the response system cannot converge to those of the drive system and synchronization is impossible.

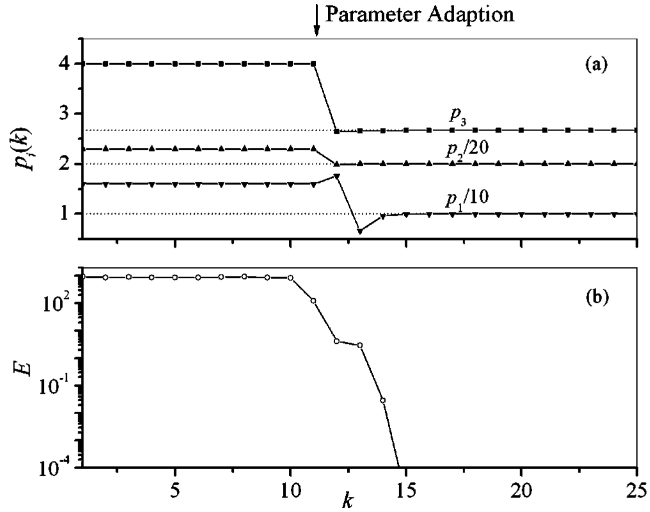

FIG. 1. (a) The convergence of the response model parameters $(\tilde{p}_1, \tilde{p}_2, \tilde{p}_3)$ to (10,40,8/3). (b) The time-averaged synchronization error, where $\tau = 0.01$, $N = 10^4$, and $\Delta = 10^{-4}$.

Parlitz [12] proposed an autosynchronization method to estimate model parameters of the Lorenz system, in which an ansatz for the parameter control loop is required. When the system is very complicated, predefining the smooth controlling forces and the parameter control loop may be compli-

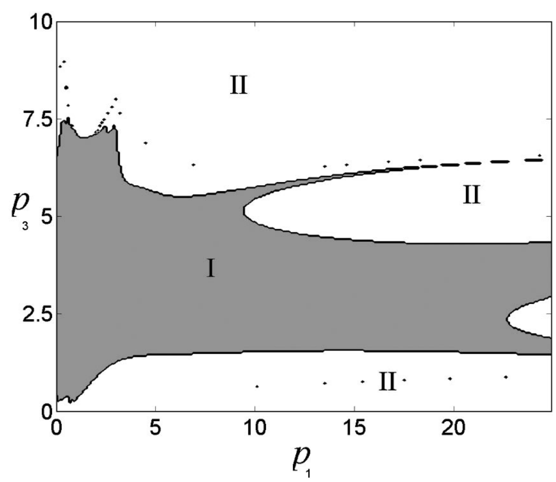

FIG. 2. The dependence of the parameter estimation results on the original values of $\tilde{p}_1$ and $\tilde{p}_3$, where $p_2 = \tilde{p}_2 = 40$. In the black region I, $\tilde{p}_1$ and $\tilde{p}_3$ converge to 10 and 8/3, respectively.

cated tasks. However, our adaption method does not require any predefined control loop. By iterating the parameter difference map, the adaptive parameter control is dynamically achieved. The robustness of this method allows us to estimate model parameters of very complex systems. In the second example, we show that this method can be applied to estimate the model parameters of a spatiotemporal chaotic system. Although, there are some other attempts, such as adaptive control [13] and random optimization [14], for estimating model parameters using chaos synchronization. The applicability of these methods to spatiotemporal chaotic systems has not yet been examined.

The coupled map lattice model is used because of its wide variety of novel and complex spatiotemporal behaviors, including spatiotemporal chaos [4,17]. The drive OCOML system with length L is represented as

$$x_1(n+1)=f(x_1(n),\mu), \quad g(n)=2\epsilon f(x_1(n),\mu),$$

$$x_i(n+1)=(1-\epsilon)f(x_i(n),\mu)+\epsilon f(x_{i-1}(n),\mu), \qquad (8)$$

and we let the response system be

$$y_1(n+1)=(1-2\epsilon')f(y_1(n),\mu')+g(n),$$

$$y_i(n+1)=(1-\epsilon')f(y_i(n),\mu')+\epsilon' f(y_{i-1}(n),\mu'), \quad (9)$$

where the subscript $i=1,2,\ldots,L$ denotes the lattice site index, and n is the time index. ϵ and ϵ' are the coupling constants. μ and μ' are the nonlinear parameters of the nonlinear function $f(x)$. For $\mu=\mu'$ and $\epsilon=\epsilon'$, the decomposition given by $g(n)=2\epsilon f(x_1(n),\mu)$ leads to the synchronization condition $|(1-2\epsilon)f_{x_1}|\ll 1$ and $|(1-\epsilon)f_{x_i}|\ll 1$ [6]. By letting $L=100$, $f(x)=1-\mu x^2$, $\mu=1.9$, and $\epsilon=0.4$, we show the spatiotemporal chaotic pattern of the drive OCOML system in the top curve of Fig. 3. A different spatiotemporal pattern of the response system with $\mu'=1.8$ and $\epsilon'=0.3$ is shown in the middle curve of Fig. 3. When parameter adaption with $\Delta=10^{-4}$ and $N=10^4$ is applied, the response parameters converge to $\mu'=1.9$ and $\epsilon'=0.4$. The spatiotemporal chaotic pattern of the drive system is reconstructed in the response system without error (see the bottom curve of Fig. 3). Parameter estimation and spatiotemporal chaos synchronization are achieved even when the original values of the response parameters are chosen liberally within a certain range. Figure 4 shows the effects of the original values of μ' and ϵ' on parameter adaption. When the original values of μ' and ϵ' are within the black region I, their asymptotic results go to $\mu=1.9$ and $\epsilon=0.4$, and synchronization is achieved. Therefore, this method of parameter adaption allows us to estimate model parameters of a spatiotemporal chaotic system. However, within the white region II, Eq. (7) has an unstable zero solution, and thus the response parameters do not converge to the drive parameters and the synchronization of spatiotemporal chaos cannot be achieved.

In the third example, we investigate a biomedical application of this parameter adaption method to a vocal fold system. Vocal fold model plays an important role in studying voice physiology. Estimation of model parameters is a clinically significant but difficult task. Trevisan *et al.* [18] applied

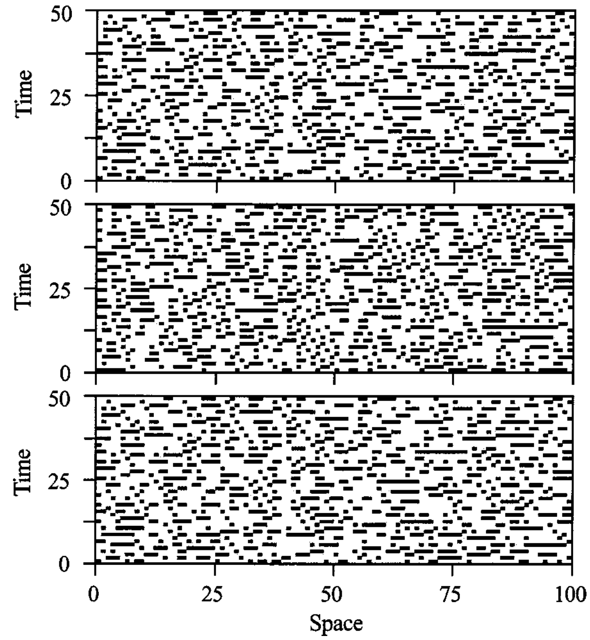

FIG. 3. The spatiotemporal chaos in the drive system with $\mu=1.9$ and $\epsilon=0.4$ (top), the response system with the original parameters $\mu'=1.8$ and $\epsilon'=0.3$ (middle), and the response system with the estimated parameters $\mu'=1.9$ and $\epsilon'=0.4$ (bottom). Pixels are black when $x_i(n)\geqslant 0.75$ and otherwise white [4].

a genetic algorithm to extract the parameters of one-mass vocal fold model. However, one-mass model is an oversimplification of the vocal fold dynamics since two or more modes are needed to capture vocal fold vibration [19]. In particular, whether the method by Trevisan *et al.* is capable for chaotic model of vocal folds had not been determined. To study chaotic vibrations of vocal folds, we applied a two-mass model proposed by Steinecke and Herzel [20]. The systematic diagram is illustrated in Fig. 5. This two-mass model of vocal folds combines nonlinear biomechanical and aerodynamic effects. Asymmetric tension parameter Q repre-

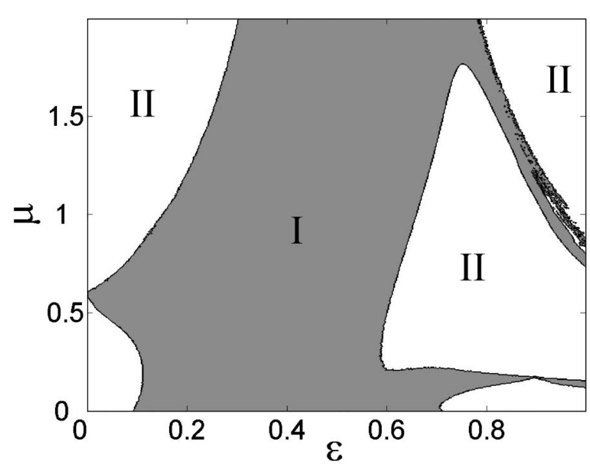

FIG. 4. The dependence of the parameter estimation results on the original values of μ' and ϵ'. In the black region I, the applied parameter adaption leads to the convergence of μ' and ϵ' to 1.9 and 0.4, respectively.

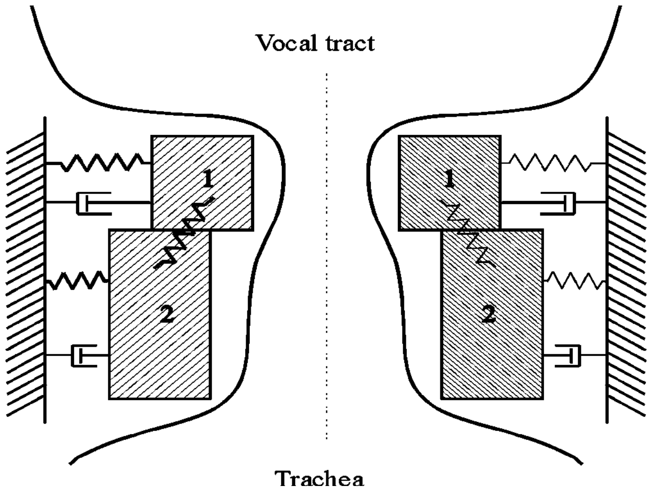

FIG. 5. The schematic diagram of the vocal fold model.

sents an important parameter for assessing voice disorders resulting from vocal fold paralyses. When the asymmetric tension parameter Q deviates from the normal value 1, chaos associated with disordered voice in patients with laryngeal paralysis may occur. Although the clinical importance of the asymmetric tension parameter, previous study based on linear approximation cannot accurately measure Q in chaotic models [21,22]. Here we apply chaos synchronization and the parameter adaption method to estimate asymmetric tension parameter Q of the two-mass model whose dynamics satisfies [20]

$$m_{i\alpha}\ddot{x}_{i\alpha}+r_{i\alpha}\dot{x}_{i\alpha}+k_{i\alpha}x_{i\alpha}+\Theta(-a_i)c_{i\alpha}\frac{a_i}{2l}+k_{c\alpha}(x_{i\alpha}-x_{j\alpha}) = P_i l d_i, \tag{10}$$

where the pressures are

$$P_1=P_s\left[1-\Theta(a_{min})\left(\frac{a_{min}}{a_1}\right)^2\right]\Theta(a_1),$$

$$P_2=0.$$

The function

$$\Theta(x)=\begin{cases}\tanh(50x/x_0), & x>0\\ 0, & x\leqslant 0\end{cases}$$

describes the vocal fold collision. The minimal glottal area is $a_{min}=\min(a_{1l},a_{2l})+\min(a_{1r},a_{2r})$, and $a_i=a_{0i}+l(x_{il}+x_{ir})=a_{il}+a_{ir}$ describes the lower and upper glottal area, $x_{i\alpha}$ is the oscillation amplitude. The indices $i,j=1,2$ denote the upper and lower masses. $\alpha=l,r$ denotes left and right vocal folds. $m_{i\alpha}$, $r_{i\alpha}$, $k_{i\alpha}$, $k_{c\alpha}$, and $c_{i\alpha}$ denote masses, damping ratio, stiffness, coupling stiffness, and additional stiffness, respectively. Q describing the tension imbalance in unilateral superior nerve paralysis satisfies $m_{ir}=m_{il}/Q$, $k_{ir}=Qk_{il}$, $k_{cr}=Qk_{cl}$, and $c_{ir}=Qc_{il}$. When $Q=0.529$, a chaotic vibration is produced for the standard parameter configuration [20]: $m_{1l}=0.125$, $m_{1r}=m_{1l}/Q$, $m_{2l}=0.025$, $m_{2r}=m_{2l}/Q$, $r_{1l}=r_{1r}=r_{2l}=r_{2r}=0.02$, $k_{1l}=0.08$, $k_{1r}=Qk_{1l}$, $k_{2l}=0.008$, $k_{2r}=Qk_{2l}$, $c_{1l}=3k_{1l}$, $c_{2l}=3k_{2l}$, $c_{1r}=Qc_{1l}$, $c_{2r}=Qc_{2l}$, $k_{cl}=0.025$, $k_{cr}=Qk_{cl}$, $d_1=0.25$, $d_2=0.05$, $a_{01}=a_{02}=0.05$, $l=1.4$, and $P_s=0.015$. Applying the method suggested by Henon [23] to the vocal fold model [24], a Poincare section for $x_{1l}=0$ shows a chaotic attractor in Fig. 6(a). To measure Q, a feedback method of chaos synchronization [3] is applied to the response system,

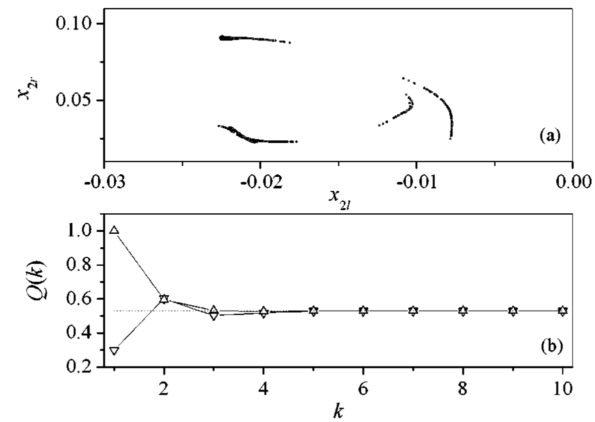

FIG. 6. (a) The chaotic attractor of the vocal fold model in the Poincare section for $x_{1l}=0$. (b) The convergence of the asymmetric tension parameter $Q(k)$ from two original values 0.3 and 1 to 0.529.

$$m'_{i\alpha}\ddot{y}_{i\alpha}+r_{i\alpha}\dot{y}_{i\alpha}+k'_{i\alpha}y_{i\alpha}+\Theta(-a_i)c'_{i\alpha}\frac{a_i}{2l}+k'_{c\alpha}(y_{i\alpha}-y_{j\alpha}) = P_i l d_i+K(x_{i\alpha}-y_{i\alpha})+K(\dot{x}_{i\alpha}-\dot{y}_{i\alpha}), \tag{11}$$

where $K=5$ is the feedback coefficient, $m'_{il}=m_{il}$, $k'_{il}=k_{il}$, $k'_{cl}=k_{cl}$, $c'_{il}=c_{il}$, $m'_{ir}=m_{il}/Q(k)$, $k'_{ir}=Q(k)k_{il}$, $k'_{cr}=Q(k)k_{cl}$, and $c'_{ir}=Q(k)c_{il}$. The iterative method of parameter adaption is applied to control the parameters $Q(k)$, $m'_{i\alpha}$, $k'_{i\alpha}$, $k'_{c\alpha}$, $c'_{i\alpha}$ of the response system. With an increase of k, Fig. 6(b) shows the convergent results of $Q(k)$ of the response system. Here, although the response system's $Q(k)$ begins from two different original values [$Q(0)=0.3$ and 1], the asymptotical values of $Q(k)$ converge to the true value 0.529. When Q(0) is within the region $0.29\leqslant Q(0)\leqslant 3.1$, the asymmetric parameter value 0.529 of the drive vocal fold model can be precisely estimated. Therefore, applying chaos synchronization and this parameter adaption method makes it practical to calculate asymmetric tension parameters in a chaotic vocal fold model.

In this paper, we have proposed an iterative scheme of parameter adaptions based on chaos synchronization. When parameter adaptions are used, two chaotic systems with an original parameter mismatch are finally synchronized and their model parameters converge to the same results. This parameter adaption method has a potential application in decoding chaotic secure communication. Previous works have suggested that by using chaos synchronization, information signals can be recovered in a response system that has the same keys or model parameters as a drive system, but cannot be decoded by a decoder with different keys [1–6]. However, using the parameter adaption method in chaotic communication, the decoder may decode the message as well as the keys of the encoder without exactly knowing the model parameters. A spatiotemporal chaotic system may not guarantee that its model parameters can avoid being decoded.

Global and local techniques [9,10] have been used to reconstruct model parameters of chaotic systems [11]. A local method in spatial extended systems has been used to predict a spatiotemporal time series [25]. However, this method involves numerous calculations, and in particular, the model parameters of spatiotemporal chaotic systems have not been estimated. We have applied a simple method to estimate the model parameters of a spatiotemporal chaotic OCOML system and synchronize two spatiotemporal chaotic systems. Finally, we have presented a biomedical application example in vocal folds. We are able to estimate the asymmetric tension parameter associated with unilateral superior nerve paralysis. In biomedical systems, a model equation can usually be specified, but direct measures of system parameters are difficult. Chaos synchronization and parameter adaptions may be valuable new methods that can be used to estimate unknown parameter values in biomedical systems.

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